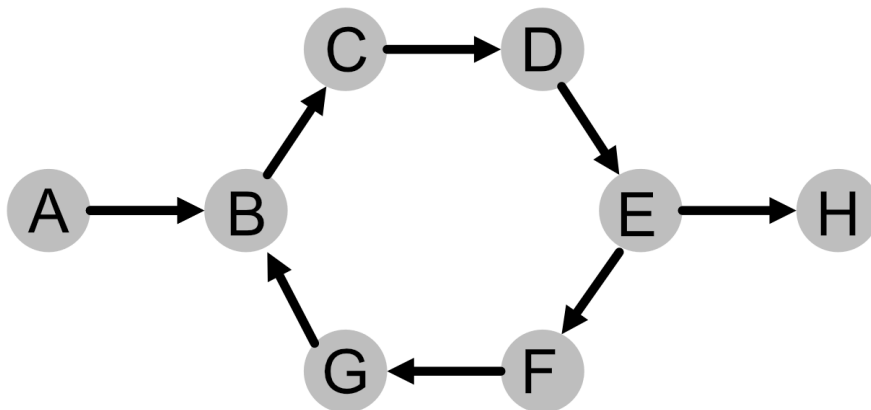


**Problem 1. Graph irreducible and aperiodic (10 points).**

Graph 1 is a directed graph that has 8 nodes and 8 directed edges.

Is this graph irreducible? If it is **not** irreducible, which single edge can we add to make it irreducible? Please list **all** the possible edges, including the self-loop if applicable. (5 points)

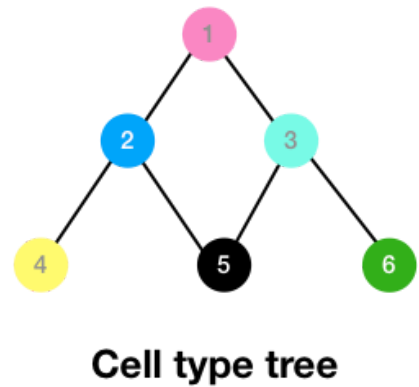
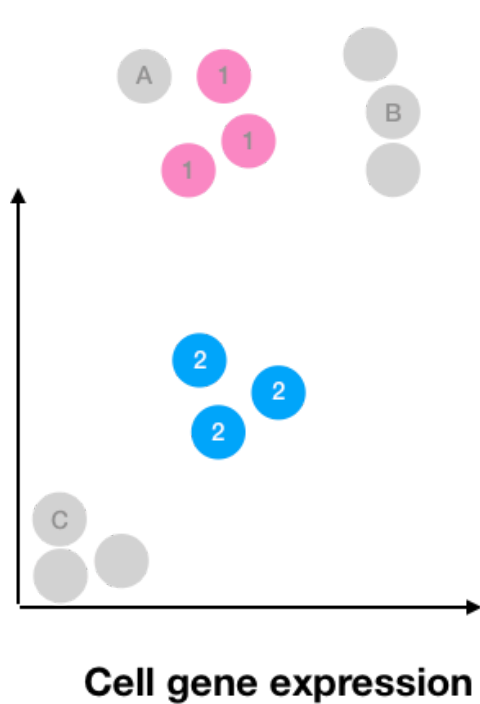
Is this graph aperiodic? If it is **not** aperiodic, which single edge can we add to make it aperiodic? Please list **all** the possible edges, including the self-loop if applicable. (5 points).



**Answer:**

**Problem 2. Tree-based cell type classification (10 points).**

We observed the expression of many cells (left panel). We know the cell types of some of them (colored in blue, pink, green). We don't know the cell types of the remaining (colored in gray). There are in total six cell types (blue, pink, cyan, yellow, green, black). Their similarities are documented in a cell type tree, which can help us do the classification (right panel). Using the tree-based classification method we introduced, what might be the cell type of A, B, C? (You only need to write down the color of A, B and C.)



**Answer:**

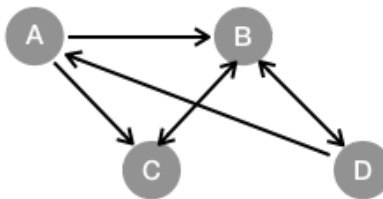
### Problem 3. Network-based mutation profile clustering (10 points).

We have talked about how the network can help us cluster patients based on their mutation profile. We used a network to do the random walk with restart on the patient mutation matrix and then clustered these stationary distributions.

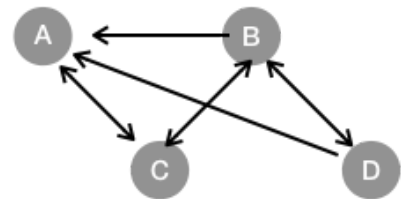
In real-world applications, we often have more than one network. If we have two different networks, how would you use both of them to do the clustering? Below is an example of the dataset. There are 5 patients, 4 genes. There are also two networks defined on the same set of genes. These two networks have different sets of edges. You don't need to do the actual clustering using two networks. A brief outline of the method would be enough.

	1	2	3	4	5
A	1	0	0	0	0
B	0	1	1	1	0
C	0	0	0	1	1
D	1	1	1	0	0

Mutation profile  
of 5 patients on 4 genes



Network 1



Network 2

**Answer:**